

Process capability monitoring chart with an application in the silicon-filler manufacturing process

K.S. Chen^a, K.T. Yu^b, S.H. Sheu^{c,*}

^a*Department of Industrial Engineering and Management, National Chin-Yi Institute of Technology, 35, Lane 215, Sec. 1, Chung san Road, Taiping, Taichung 411, Taiwan, ROC*

^b*General Education Center, Tzuchi College of Technology, 880, Sec.2, Chien-kuo Rd. Hualien, Taiwan*

^c*Department of Industrial Management, National Taiwan University of Science and Technology, #43, Sec. 4, Keelung Road, Taipei 106, Taiwan, ROC*

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Abstract

Process capability indices (PCIs) can be viewed as effective and excellent means of measuring product quality and performance, but they are not sufficient for evaluating an entire product with multiple characteristics. [Chen, K.S., Huang, M.L., Li, R.K., 2001. Process capability analysis for an entire product. *International Journal of Production Research* 39(17), 4077–4087] proposed the process capability analysis chart (PCAC), applied indices C_{pu} , C_{pl} and C_{pa} to evaluate the integrated process capability for a multi-process product. However, as noted by [Pearn, W.L., Chen, K.S., 1997. Multi-process performance analysis: a case study. *Quality Engineering* 10(1), 1–8], when we take into account the asymmetric tolerance, C_{pn} reflects the process capability more accurately and is superior to index C_{pa} . In this paper, we select C_{pn} to replace C_{pa} , and reconstruct a process capability monitoring chart (PCMC) for evaluating process potentials and performance for an entire product, which consists of smaller-the-better (with C_{pu}), larger-the-better (with C_{pl}), nominal-the-best (with C_{pn}) specifications, respectively. Meanwhile, an integrated product capability index is proposed, and the relationship between the index and the process yield of an entire product is described. A PCMC, which reasonably and accurately indicates the status of all process capabilities for the silicon-filler product, is designed for practical applications. © 2006 Elsevier B.V. All rights reserved.

Keywords: Process capability indices; Multi-process product; Process yield; Asymmetric tolerances

1. Introduction

Process capability indices (PCIs) are intended to provide single-number assessment of ability to meet specification limits on quality characteristics of interest. A larger PCI also implies a higher process

yield and a lower process expected loss. Therefore, PCIs can be viewed as effective and excellent means of measuring product quality and performance, and have been widely used in manufacturing industry. Numerical statisticians and quality engineers, such as Kane (1986), Chan et al. (1988), Choi and Owen (1990), Boyles (1991), Pearn et al. (1992), Kotz and Johnson (1993), Boyles (1994) and Spiring, 1997, have emphasized the research of PCIs to propose more precise methods of evaluating

*Corresponding author. Tel.: +886 2 27376335; fax: +886 2 27376344.

E-mail address: ykt.kevin@msa.hinet.net (S.H. Sheu).

Nomenclature

$$A = \max\{d^*(\mu - T)/D_u, d^*(T - \mu)/D_l\}$$

$$C_T \quad \text{integrated product capability index}$$

$$C_a = 1 - \max\{\mu - T/D_u, T - \mu/D_l\}$$

$$C_{dl} = (d^*/D_l) \left((\mu - \text{LSL}) / \left(3 \sqrt{\sigma^2 + A^2} \right) \right)$$

$$C_{du} = (d^*/D_u) \left((\text{USL} - \mu) / \left(3 \sqrt{\sigma^2 + A^2} \right) \right)$$

$$C_{pa} = (d^* - A)/3\sigma$$

$$C_{pl} = (\mu - \text{LSL})/3\sigma$$

$$C_{pn} = \min(C_{du}, C_{dl})$$

$$C_{pu} = (\text{USL} - \mu)/3\sigma$$

$$D_l = T - \text{LSL}$$

$$D_u = \text{USL} - T$$

PCMC process capability monitoring chart

T process target

LSL the lower specification limit

USL the upper specification limit

Z standard normal distribution

$$d^* = \min\{D_u, D_l\}$$

v_0 the critical value of the individual process capability

$\Phi(\cdot)$ the cumulative function of standard normal distribution

Φ^{-1} the inverse cumulative function of standard normal distribution

μ process mean

σ process standard deviation

process potentials and performance. However, they take account of a process with single quality characteristic only, and limits do exist when applying those indices to multiple quality characteristic products.

As noted by Bothe (1992) and Chen et al. (2001), most products with multiple characteristics could consist of numerous unilateral specifications and bilateral specifications. In fact, customers will accept products whenever all process capabilities of each characteristic satisfy preset specifications. Obviously, a single PCI cannot meet the requirements stated as above. Furthermore, another important problem is focusing on many bilateral quality characteristics with asymmetric tolerances.

For this reason, many scholars have provided some graphical methods that can monitor all the processes, whether they meet the quality level or not. First, Singhal (1991) proposed a C_{pk} multi-process performance analysis chart (C_{pk} MPPAC) to evaluate the performance of a multi-process product with symmetric bilateral specifications. Then, Pearn and Chen (1997) proposed a modification to the C_{pk} MPPAC combining the more-advanced PCIs C_{pm} to identify the problems causing the process failing to center around the target. The modified C_{pk} MPPAC provides an easy way to process improvement by comparing the locations on the chart of the processes before and after the improvement effort. Vännman and Deleryd, 1999 used the yield index S_{pk} , which was proposed by Boyles, constructed a process capability plot to define the capability of the process, called the (δ, γ) plot, where $\delta = (\mu - T)d$, and $\gamma = \sigma/d$. The (δ, γ) plot is an effective graphical method for theoretic-

cally comparing and contrasting different PCIs, and is invariable with respect to the value of the specifications. Combining Singhal's MPPAC with asymmetric PCI C_{pa} , plus considering unilateral characteristics, Chen et al. (2001) developed a process capability analysis chart (PCAC) to evaluate process potentials and performance for an entire product, which consists of smaller-the-better unilateral specifications (with C_{pu}), larger-the-better unilateral specifications (with C_{pl}), and asymmetric bilateral specifications (with C_{pa}).

Kane (1986) proposed PCIs C_{pu} and C_{pl} for measuring smaller-the-better and larger-the-better process capabilities:

$$C_{pu} = \frac{\text{USL} - \mu}{3\sigma}, \quad C_{pl} = \frac{\mu - \text{LSL}}{3\sigma},$$

where USL and LSL are the upper and lower specification limits respectively, μ is the process mean and σ is the process deviation.

For the nominal-the-best process in typical quality control applications, PCIs are used to evaluate how well the process meets specifications. A process is said to have symmetric tolerance if the target value T is the midpoint of the specification interval (LSL, USL). Although cases with symmetric tolerances are common in practical situations, they often occur in manufacturing industry. Pearn and Chen (1998) proposed a PCI C_{pa} for processes with asymmetric tolerance:

$$C_{pa} = \frac{d^* - A}{3\sigma},$$

where $A = \max\{d^*(\mu - T)/D_u, d^*(T - \mu)/D_l\}$, $D_u = \text{USL} - T$, $D_l = T - \text{LSL}$ and $d^* = \min\{D_u, D_l\}$.

Next year, Pearn et al. (1999) proposed a generalization of C_{pn} for processes with asymmetric tolerances. It may be redefined as

$$C_{pn} = \frac{d^* - A}{3\sqrt{\sigma^2 + A^2}}$$

As noted by Pearn et al. (1999), the generalization takes into account the asymmetry of tolerance, which reflects the process capability more accurately and is superior to index C_{pa} . Thus, in this study, we select C_{pn} to replace C_{pa} and reconstruct a process capability monitoring chart (PCMC), which extends PCIs application from assessing the process capability for a single quality characteristic to evaluating the integrated product capability for a multi-process product. Meanwhile, an integrated product capability index is proposed, and the relationship between the index and the process yield of an entire product is described.

In addition, Chen et al. (2001) indicated clearly that index C_a measures the relative distance of the shift from process mean to preset target. It is defined as

$$C_a = 1 - \max\left\{\frac{\mu - T}{D_u}, \frac{T - \mu}{D_l}\right\}$$

The definition of relative distance is $(\mu - T)/D_u$ or $(T - \mu)/D_l$. Equal relative distances result in same values of C_a . Obviously, the index C_a , which can indicate the process accuracy and process loss, is considered in the PCMC. We derive some results between the indices C_{pn} and C_a . Once the quality level is decided, we make use of the results and mark the process capability zone with bold lines on the PCMC for checking whether the process capabilities satisfy preset level or not.

PCIs and the PCMC are used to evaluate how well the processes meet specifications, and assess the integrated process capability for a multi-process product. Finally, the proposed PCMC is applied to a manufacturing process of a silicon-filler product, illustrating how the PCMC can be employed to evaluate the process capabilities of an entire product with numerous quality characteristics.

2. PCMC

Since index C_{pn} reflects the process capability more accurately and is superior to index C_{pa} , this study replaces index C_{pa} with index C_{pn} . Thus, we reconstruct a PCMC, which is composed of three indices C_{pn} , C_{pu} and C_{pl} . They reasonably and

accurately indicate the status of all process capabilities for an entire product. The definition of C_{pn} can be rewritten as $C_{pn} = \min\{C_{du}, C_{dl}\}$, where

$$C_{du} = \left(\frac{d^*}{D_u}\right) \left(\frac{USL - \mu}{3\sqrt{\sigma^2 + A^2}}\right),$$

$$C_{dl} = \left(\frac{d^*}{D_l}\right) \left(\frac{\mu - LSL}{3\sqrt{\sigma^2 + A^2}}\right).$$

The PCMC characterizes not only the process capabilities with symmetric and asymmetric tolerances on the dimension space, but also the process capabilities with smaller-the-better and larger-the-better types on the X - and Y -axes, respectively. X -axis represents simultaneously C_{du} for the nominal-the-best process, and C_{pu} for the smaller-the-better process. Similarly, Y -axis represents simultaneously C_{dl} for the nominal-the-best process, and C_{pl} for the larger-the-better process. Axes X and Y construct the PCMC as shown in Fig. 1.

According to the loss function stated in the Taguchi method, the closer the process mean to the process target implies better quality and fewer losses. Conversely, the further the process mean from the process target implies worse process capabilities. Likewise, keeping the process on-target is crucial. A few subsidiary lines of C_a can be added to the PCMC for controlling precisely the process shifts. According to Chen et al. (2001), let $C_a = (1 - 1/a)$, then the values of μ are $[T + (1/a)D_u]$ and $[T - (1/a)D_l]$ for each C_a . The slope of the corresponding subsidiary line is $(a + 1)/(a - 1)$ when the process mean is greater than the process target, and the slope of the corresponding

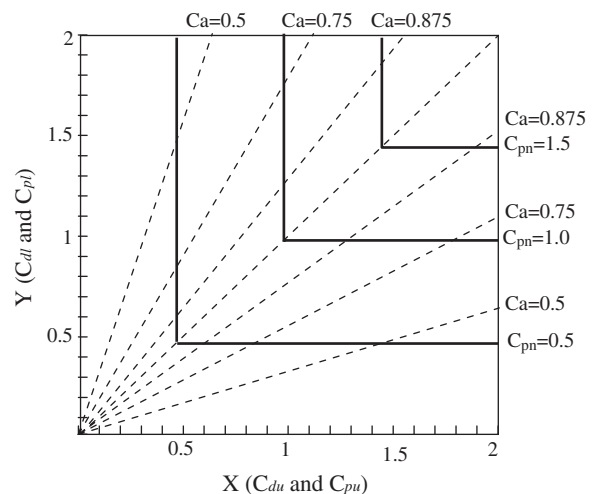


Fig. 1. Process capability monitoring chart.

subsidiary line is $(a-1)/(a+1)$ when the process mean is smaller than the process target, shown in Fig. 1. Obviously, C_a indicates the accuracy of the process. In general, C_a cannot be too small, since a smaller C_a implies that the process mean shifts further away from the process target, thus resulting in much process losses.

2.1. Evaluation of process capability

To evaluate one product with multiple characteristics, assume that the number of nominal-the-best, smaller-the-better and larger-the-better characteristics are c_n , c_u and c_l , which are evaluated, respectively, by

$$C_{pnj} = \frac{d_j^* - A_j}{3\sqrt{\sigma_j^2 + A_j^2}}, \quad \text{for } j = 1, 2, \dots, c_n,$$

$$C_{puj} = \frac{USL_j - \mu_j}{3\sigma_j}, \quad \text{for } j = 1, 2, \dots, c_u,$$

$$C_{plj} = \frac{\mu_j - LSL_j}{3\sigma_j}, \quad \text{for } j = 1, 2, \dots, c_l.$$

We derive the formula regarding C_{pn} and process yield (P_n) as follows. Under normal assumptions, let X be the random number of process mean and Z be the standard normal distribution:

$$\begin{aligned} P_n &= \Pr(LSL \leq X \leq USL) \\ &= \Pr\left(\frac{LSL - \mu}{\sigma} \leq \frac{X - \mu}{\sigma} \leq \frac{USL - \mu}{\sigma}\right) \\ &\geq \Pr\left(\frac{d^*}{\sqrt{\sigma^2 + A^2}} \frac{LSL - \mu}{D_l} \leq Z \leq \frac{d^*}{\sqrt{\sigma^2 + A^2}} \frac{USL - \mu}{D_u}\right) \\ &= \Pr(-3C_{dl} \leq Z \leq 3C_{du}) \\ &\geq \Pr(-3C_{pn} \leq Z \leq 3C_{pn}) \\ &= 2\Phi(3C_{pn}) - 1. \end{aligned}$$

That is, process yield $P_{nj} \geq 2\Phi(3C_{pnj}) - 1$, $j = 1, 2, \dots, c_n$.

On the other hand, the unilateral specification indices C_{puj} and C_{plj} hold the one-to-one mathematical relation with the process yield (P_{ij}) under normal assumptions. The formula for the larger (smaller)-the-better process can be described as

$$P_{ij} = \Phi(3C_{p_{ij}}), \quad i \in \{u, l\}, \quad \text{for } j = 1, \dots, c_i.$$

The above can be summarized as

$$P_{ij} \geq 2\Phi(3C_{p_{ij}}) - 1, \quad \text{for } i \in S = \{n, u, l\}, \quad j = 1, \dots, c_i.$$

Obviously, the larger the process capability, the higher is the process yield. We intend to define a product capability index (C_T) to express the integrated process capability of an entire product:

$$C_T = \left(\frac{1}{3}\right) \Phi^{-1} \left(\left[\left(\prod_{i \in S} \prod_{j=1}^{c_i} [2\Phi(3C_{p_{ij}}) - 1] \right) + 1 \right] / 2 \right),$$

especially when $C_T = v$, $\prod_{i \in S} \prod_{j=1}^{c_i} [2\Phi(3C_{p_{ij}}) - 1] = 2\Phi(3v) - 1$.

Assuming process yields for each characteristic are independent, the entire product yield (P_T) can be described as:

$$\begin{aligned} P_T &= \prod_{i \in S} \prod_{j=1}^{c_i} (P_{ij}) \geq \prod_{i \in S} \prod_{j=1}^{c_i} [2\Phi(3C_{p_{ij}}) - 1] \\ &= 2\Phi(3v) - 1. \end{aligned}$$

There exists a mathematical mapping relationship between the product capability index (C_T) and the process yield of an entire product (P_T). A greater product capability index (C_T) corresponds to a higher process yield of the entire product (P_T). For instance, when the product capability index $C_T = 1.0$ and 1.33 , the corresponding process yields of the entire product (P_T) equal 99.73% and 99.99%, respectively.

Based on the above analysis, the process yield of the entire product (P_T) is definitely lower than any individual process yield. Similarly, when the entire product capability is preset to meet the required level (v), the individual process capability should be greater than the preset standard (v_0), e.g. if $C_T \geq v$, then the individual process capability $C_{p_{ij}} \geq v_0$. Especially, when the preset minimum values of process capabilities for each characteristic are equal, then the critical value v_0 for individual process capability can be attained:

$$v_0 = \left(\frac{1}{3}\right) \Phi^{-1} \left(\frac{\sqrt{2\Phi(3v) - 1} + 1}{2} \right),$$

where $c = c_n + c_u + c_l$.

Pearn and Chen (1997) proposed five quality conditions, and the corresponding values of v_0 , as shown in Table 1.

Table 1
Five quality conditions

Quality condition	Value of v_0
Inadequate	$v_0 < 1.00$
Capable	$1.00 \leq v_0 < 1.33$
Satisfactory	$1.33 \leq v_0 < 1.50$
Excellent	$1.50 \leq v_0 < 2.00$
Super	$2.00 \leq v_0$

2.2. Process capability zone

Based on the description in Section 2.1, when we consider that $C_{pn} \geq v_0$, since

$$C_{pn} = \frac{d^* - A}{3\sqrt{\sigma^2 + A^2}} = \frac{(1 - A/d^*)}{3\sqrt{(\sigma/d^*)^2 + (A/d^*)^2}}$$

$$= \frac{C_a}{3\sqrt{(\sigma/d^*)^2 + (1 - C_a)^2}} \geq v_0,$$

$$\frac{C_a}{3(1 - C_a)} \geq v_0,$$

$$C_a \geq \frac{3v_0}{3v_0 + 1}.$$

Obviously, when C_{pn} becomes greater, C_a also becomes larger. We can find the relative distance getting smaller for each C_a . We consider the minimum $C_a = 3v_0/(3v_0 + 1) = 1 - (1/a)$. The slope of the corresponding subsidiary line is $(a + 1)/(a - 1) = (3v_0 + 2)/3v_0$ when the process mean is greater than the process target, and the slope of the corresponding subsidiary line is $(a - 1)/(a + 1) = 3v_0/(3v_0 + 2)$ when the process mean is smaller than the process target. Briefly, if the individual process capability $C_{pij} \geq v_0$, then we can mark the process capability zone S_{v_0} with bold lines, as shown in Fig. 2, where

$$S_{v_0} = \left\{ (x, y) \mid \frac{3v_0}{3v_0 + 2} \leq \frac{y}{x} \leq \frac{3v_0 + 2}{3v_0}, x \geq v_0, y \geq v_0 \right\}.$$

The upper boundary point (UP) of S_{v_0} places at $(v_0, v_0 + (2/3))$ and the lower boundary point (LP) of S_{v_0} places at $(v_0 + (2/3), v_0)$.

On the whole, the minimum PCIs v_0 is specified when C_T and the number of individual process characteristics c are selected. Furthermore, we calculate the minimum $C_a = 3v_0/(3v_0 + 1)$, $UP(v_0, v_0 + (2/3))$ and $LP(v_0 + (2/3), v_0)$. The process capability zone will be marked with bold

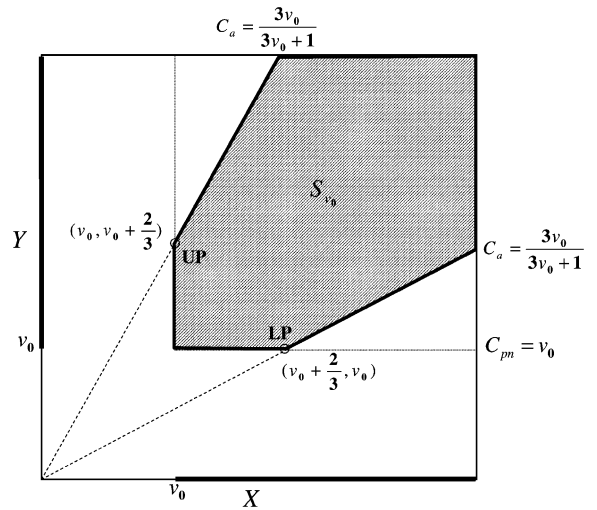


Fig. 2. Process capability zone.



Fig. 3. Silicon-filler products.

lines on the PCMC according to the minimum individual process capability v_0 and the maximum process shift C_a . After that, we utilize the PCMC to check whether the indices are located in the zone or not.

3. Illustrative example

This example involves a polymerization process in silicon-filler manufacturing factory. The silicon filler is employed in a variety of products, which includes mechanical parts of mobile phone, flexible printed circuit board (FPCB) module and auto parts (as shown in Fig. 3). It is also a functionally critical part for the silicon compound RC50 used in the computer peripheral assembly.

The key process characteristics of the silicon-filler products include: (1) density, plasticity, shrinkage, hardness and hardness change which are nominal-the-best process characteristics, (2) tensile strength, elongation, tear strength and rebound which are larger-the-better process

Table 2
Specifications and capability indices for 15 process characteristics

Process	Process characteristic	LSL	T	USL	μ	σ	C_a	X-axis	Y-axis	C_{pn}
N1	Density (g/cm ³)	1.140	1.146	1.150	1.145	0.001	0.833	1.600	1.067	1.067
N2	Plasticity (point)	228.000	232.000	238.000	233.000	1.200	0.833	0.809	1.214	0.809
N3	Shrinkage (%)	3.400	3.500	3.600	3.520	0.004	0.800	1.307	1.961	1.307
N4	Hardness (%)	51.000	52.000	53.000	51.700	0.350	0.700	0.940	0.506	0.506
N5	Hardness change (%)	0.000	0.500	1.000	0.470	0.100	0.940	1.692	1.501	1.501
L1	Tensile strength(kg/cm ²)	7.500			8.000	0.100			1.667	
L2	Elongation (%)	350.000			365.000	3.800			1.316	
L3	Tear strength (kg/cm)	6.500			7.800	0.400			1.083	
L4	Rebound (%)	70.000			82.000	2.600			1.538	
S1	Compression (%)			8.000	6.000	1.000		0.667		
S2	Yellowing (point)			-6.000	-8.500	0.500		1.667		
S3	Reaction time (s)			185.000	180.000	1.200		1.389		
S4	Plasticity rate (s)			30.000	25.000	1.400		1.190		
S5	Release force (g)			260.000	248.000	3.200		1.250		
S6	Adhesion (g)			6.000	5.000	0.4000		0.833		

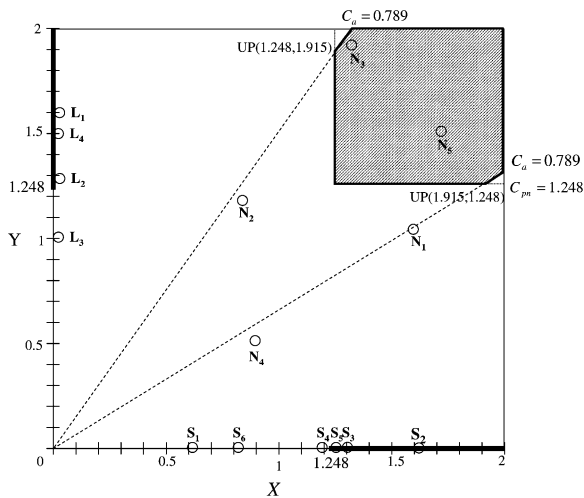


Fig. 4. Process capability zone.

characteristics, and (3) compression, yellowing, reaction time, plasticity rate, release force and adhesion which are smaller-the-better process characteristics. Table 2 displays process specifications and capability indices for 15 process characteristics about the process of the silicon-filler products.

Assume that the entire process capability is preset to exceed one ($C_T \geq v = 1$), the minimum individual process capability is $v_0 = \Phi^{-1} [\sqrt{15} \sqrt{2\Phi(3) - 1} + 1] / 2 / 3 = 1.248$, where $c = 15$. Furthermore, the process loss is considered, then we get $C_a \geq 0.789$ and calculate UP(1.248,1.915) and

LP(1.915,1.248). The process capability zone is marked with bold lines, as shown in Fig. 4.

Among the 15 process characteristics, seven PCIs, N1, N2, N4, L3, S1, S4 and S6 are not located within the process capability zone. Some actions must be taken for engineers to reinforce the quality level by shifting the process mean to target and reducing the process variation. Under cost considerations, all indices have to be brought back within the process capability zone, and even located near the diagonal for the nominal-the-best characteristic processes.

4. Conclusions

For monitoring the process capabilities of a multi-process product, we reconstruct an effective and efficient method via the process capability monitoring chart (PCMC), which not only retains the merits of PCAC, but also replaces the process capability index (PCI) C_{pa} with C_{pn} . The PCI $C_{pn} = \min\{C_{du}, C_{dl}\}$ is a superior index than C_{pa} , especially for evaluating the process capability with asymmetric bilateral specifications. In the PCMC, X-axis represents simultaneously C_{du} for the nominal-the-best process, and C_{pu} for the smaller-the-better process; similarly, Y-axis represents simultaneously C_{dl} for the nominal-the-best process, and C_{pl} for the larger-the-better process. Moreover, when the process loss is considered, we derive some results between the indices C_{pn} and C_a , and then we

can mark process capability zone with bold lines on the PCMC for checking whether the process capabilities satisfy preset level or not. Furthermore, quality improvement actions are taken with respect to unsatisfactory process to enhance the entire process capability. The PCMC interprets multi-characteristics process capabilities and distinguishes the process precision and accuracy with respect to the locations of the PCIs. So, it is a useful and simple tool for engineers to evaluate and provide the chances of continuous improvement on manufacturing process.

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